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"CONCERNING A METHOD FOR FINDING A PARTICULAR INTEGRAL." NOTE ON PROFESSOR COBLE'S ARTICLE.[1]

By HENRY P. MANNING, Brown University.

The method of undetermined coefficients for obtaining the particular integral in certain cases of linear differential equations is much easier and shorter than the other methods given in our text-books, except perhaps some of the mysterious symbolic methods, and it is at least as short as these. Professor Coble expresses the opinion[2] that Cohen was the first to publish it, and this may be true so far as English and American texts are concerned. Neither Johnson, nor Murray, nor Forsyth mention it. The method, however, is to be found in the second edition of the third volume of Jordan's *Cours d'Analyse*[3] which appeared some years earlier. Cohen explains the theory a little differently, but Jordan's explanation, though brief, is very clear and much simpler than Cohen's. It is so simple, in fact, that with a little elaboration it can be taught to undergraduate academic and engineering students, even when not in their books, and it can be applied not only to forms of second member that occur most frequently when the equations have constant coefficients, but also to cases of the so-called homogeneous or Cauchy equations, and to some other types that occur. It is analogous to the method of differentiation employed for certain equations that are not linear. From a given linear differential equation is obtained an equation of higher order with the second member zero, and, when the complementary function of this equation can be obtained, that part which does not belong to the complementary function of the given equation is determined by substitution so as to satisfy it as a particular integral. In practice we usually know the form of the result and have only to substitute and determine the constant coefficients.[4]

RECENT PUBLICATIONS.

REVIEW.

Analytic Geometry. By EDWIN S. CRAWLEY and HENRY B. EVANS. Philadelphia, E. S. Crawley, 1918. 12mo. 14 + 239 pages. Price, $1.60.

This book begins with seven preliminary pages given to review formulas from Algebra and Trigonometry and a set of tables, one of the three place natural values of the trigonometric functions of angles measured in radians and degrees for each degree from 0^0 to 90^0; another of three place common logarithms of numbers from 10 to 99; another of two place Napierian logarithms of numbers from 0 to 10.9; and two tables for Napierian anti-logarithms, one for positive and the other for negative powers of e; and also a table of square roots of numbers from 0 to 99 to two decimal places. These tables precede a text of twelve chapters (235 pages) of which the first eleven are devoted to plane analytic geometry and the twelfth to an "Extension of coördinate geometry to some space problems." The text is followed by an appendix (four pages) wherein are sketched twenty of the ordinary curves "for reference," used in analytic geometry and calculus.

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[1] In this MONTHLY, January, 1919, pages 12–15.

[2] In a footnote on page 13.

[3] C. Jordan, *Cours d'Analyse de l'Ecole Polytechnique,* Tome 3, 2d ed., Paris, 1896; arts. 132–133. pp. 160–163.

[4] C. E. Love explains this method briefly (*Calculus,* New York, 1916, pp. 313–315).

The subject matter treated is that of the text books now in common use with the addition of some elementary discussion of imaginary elements in geometry. The imaginary unit is introduced on page *viii* of the Introduction: " $\sqrt{-1}$ is called the imaginary unit and is usually denoted by the letter i."

On pages 20–22 in connection with a discussion of the curve $4x^2 + 9y^2 = 36$, imaginary points on the curve are defined as points which cannot be constructed but which correspond to pairs of numbers (one of them being imaginary in the illustrative example) satisfying the equation. The text does not state whether or not both of the numbers may be imaginary.

On page 30 we find: "*Definition. An equation which is satisfied by only imaginary values of the variable, or by at most a finite number of real values, is called the equation of an* **IMAGINARY LOCUS**." Three types of equations fall under this classification: "(*a*) Equations one or more of whose coefficients are imaginary numbers; (*b*) Equations all of whose coefficients are real, but which are not satisfied by any real values of the variables; and (*c*) Equations all of whose coefficients are real, but which can be satisfied by only a limited number of pairs of real values of x and y." On pages 58–60 in the chapter on the straight line, under the head of "Imaginary Points and Lines," are the definition: "*Two imaginary points whose coordinates differ only in the signs of the imaginary parts are called* **CONJUGATE IMAGINARY POINTS**," and the theorems: "*If an imaginary point lies on a real line $Ax + By + C = 0$, then the conjugate imaginary point lies on the same line*"—"*On every imaginary line there is one and only one real point*,"—and "*A pair of conjugate imaginary lines meet in a real point.*" On page 67 the imaginary circle is defined as the locus of $x^2 + y^2 + Lx + My + N = 0$ when $N > 1/4(L^2 + M^2)$.

These definitions and theorems have a few exercises following them and depending upon them. But the authors seem to have made no further use of the imaginary number in its geometric interpretation. While there is an advantage in including this material for the purpose of keeping the meaning of the imaginary number before the student, it is very doubtful if this advantage is sufficient to make up for the difficulty thereby added to the course, particularly since the greatest use of the material cannot come into play until the student reaches the study of the complex variable. The discussion is so brief that there is no mention of even the circular points at infinity. With such a brief treatment of imaginaries the student is bound to wonder what they were put in the book for. Why introduce them if they are not to be used? The preface explains by saying that "owing to the increasing use of the imaginary and its growing importance to the student of pure and applied mathematics some elementary discussion of imaginary elements in geometry has been included, which the authors believe will be of value in accustoming the student to look upon the complex number as a useful member of the number system." The reviewer doubts that the brief treatment in the book really teaches any of the *uses* of the imaginary number.

The arrangement and form of presentation of the subject matter are planned to lessen the difficulties which confront the student in his transition from elementary algebra and geometry, where memory has too often been sufficient for his needs, to a subject wherein the interpretation of equations and loci and the understanding of mathematical relations is necessary to any sort of mastery of the work. This transition marks the end of mathematical interest to a large number of students each year and the text before us as an attempt to lessen these transition difficulties seems entirely justified in spite of the fact that analytic geometries are already so numerous. The treatment generally followed in the text differs from the customary form of presentation chiefly in the putting in the form of a problem much of the material which is usually put in the form of a theorem. Thus in the chapter on the straight line the book first sets down on page 40 "Elementary geometry teaches that a straight line is determined by two conditions properly chosen." This is followed by the "*Problem. To derive the equation of a straight line in terms of the coördinates of two given points on the line.*" All the standard forms of the equation of the straight line are thus obtained as "problems." Later in the book we find the

"*Problem. To find the equation of the ellipse referred to rectangular axes with the principal axis for x axis and the origin at the center of the ellipse.*" The putting of such material in problem form makes the student feel that he is being let into the secret of the methods by which analytic geometry was built up. It is often discouraging to the student to be shown a finished demonstration without any clue to the original method of its derivation.

Especially worthy of note is Chapter XI (14 pages) in which the authors present "Empirical equations" in two cases, first, when the law of the curve is given, and second, when the law of the curve is not given. This topic is presented by means of illustrative examples based upon experimental data. The first problem is to determine the equation representing the relation of elongation to load in an actual piece of steel. Here it is assumed known that the elongation is proportional to the load and therefore that the relation is linear, so that the locus is $y = mx + b$ where y is the load and x the elongation. It remains to choose two points determined by experimental data through which a line shall pass containing as many points as possible and leaving the points not on the line equally distributed above and below it. The two chosen points serve to determine the m and b of the equation.

In the second problem, when the law is not given, the points determined from experimental data are first plotted and an equation whose locus closely approximates the true locus is chosen. The equation suggested for general use is

$$y = a + bx + cx^2 + dx^3 + \cdots$$

where a, b, c, etc., are determined from the equation by giving x and y the values obtained by experiment.

This process, now commonly found in text books on analytic geometry, has a definite application in experimental sciences and ought to be given if the time is available. It furnishes also a convenient setting for a discussion of continuity.

The last chapter of the book (31 pages) is a successful adaptation of the conceptions and equations of analytic geometry of three dimensions to the needs of the student in his first course in calculus without attempting to develop fully the subject of solid analytic geometry.

The authors have not seen fit to introduce any formal treatment of limits or continuity, although they define a tangent as the limiting position of a secant and suggest the notion of continuity in the chapter on "Empirical equations." It may be a debatable question how much of this should be given in a first course. Some teachers would be satisfied with the loose notions of elementary geometry while others would demand the precise ϵ, δ definitions, but it does seem that theories so involved in analytic geometry should be formally mentioned, and an attempt made to give to the student conceptions as accurate as his ability to understand permits.

On pages 53–56 the authors discuss the "Perpendicular distance to a given point from a given line." The discussion is based upon the unnecessary Hesse normal form of the equation of the straight line. Professor Maxime Bôcher, in his *Plane Analytic Geometry* (New York, 1915), first obtains the formula on pages 38 and 39; and on page 43 he sets up the Hesse normal form in fine print. In this MONTHLY, December, 1917, page 476, Mr. R. M. Mathews finds the distance formula without making use of the Hesse normal form. Similarly, in this MONTHLY for April, 1918, page 181, Mr. H. T. Burgess comments favorably upon the derivation of Mr. Mathews, gives still another derivation, and states that "Hesse's normal form in this connection may well be relegated to Professor Miller's collection of 'Obsoletes.'"[1]

The book is well supplied with carefully selected exercises graded into "Normal exercises" and "General exercises." On the whole the book is a very teachable class room text which should enable the student to grasp the subject matter and use it successfully in the many well selected problems of the text and in other elementary problems of analytic geometry.

Strange to say, the book has no index. This seems inexcusable in a modern text.

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NOTES.

Simplified navigation for ships and aircraft. A text book based upon the Saint Hilaire method. By C. L. POOR. New York, Century, 1918. 12mo. 18 + 126 pp. + 1 plate ["The line of position computer"]. Price $1.50.

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[1] See also Maxime Bôcher, "Concerning Direction Cosines and Hesse's Normal Form." AMERICAN MATHEMATICAL MONTHLY, September, 1918, pages 308–10.